**Shyama Prasad Mukherji College**

**Teaching Plan**

**Course and Year: B.Sc. (H) 2022-23**

**Semester: V**

**Paper: Metric Spaces**

**Taught individually or shared: Shared**

**Faculty: Mrs Alpana Rastogi & Mrs. Neeru Jain**

**No. of Classes** (per week)**: 5 Lecture + 1 Tutorial**

**Course Objectives:** Up to this stage, students do study the concepts of analysis which evidently rely on the notion of distance. In this course, the objective is to develop the usual idea of distance into an abstract form on any set of objects, maintaining its inherent characteristics, and the resulting consequences.

**Course Learning Outcomes:** The course will enable the students to:

1. Learn various natural and abstract formulations of distance on the sets of usual or unusual entities. Become aware one such formulations leading to metric spaces.
2. Analyse how a theory advances from a particular frame to a general frame.
3. Appreciate the mathematical understanding of various geometrical concepts, viz. balls or connected sets etc. in an abstract setting.
4. Know about one of the beautiful results in analysis – Banach fixed point theorem, whose far-reaching consequences have resulted into an independent branch of study in analysis, known as the fixed point theory.
5. Learn about the two important topological properties, namely connectedness and compactness of metric spaces.

### Unit 1: Basic Concepts

Metric spaces**:** Definition and examples, Sequences in metric spaces, Cauchy sequences, Complete metric space.

### Unit 2: Topology of Metric Spaces

Open and closed ball, Neighborhood, Open set, Interior of a set, Limit point of a set, Derived set, Closed set, Closure of a set, Diameter of a set, Cantor’s theorem, Subspaces, Dense set.

### Unit 3: Continuity & Uniform Continuity in Metric Spaces

Continuous mappings, Sequential criterion and other characterizations of continuity, Uniform continuity, Homeomorphism, Contraction mapping, Banach fixed point theorem.

### Unit 4: Connectedness and Compactness

Connectedness, Connected subsets of ***ℝ***, Connectedness and continuous mappings, Compactness, Compactness and boundedness, Continuous functions on compact spaces.

**Time line of Semester: 20th July2021-15th November 2021**

**Teaching Plan**

**Week 1:** Definition of metric space, Illustration using the usual metric on R, Euclidean and max

metric on R2 , Euclidean and max metric on Rn, Discrete metric, Sup metric on B(*S*)

and C[*a*, *b*], Integral metric on C[*a*, *b*].

**Week 2:** Sequences in metric space, Definition of limit of a sequence, Illustration through

examples, Cauchy sequences.

**Week 3:** Definition of complete metric spaces, Illustration through examples.

**Week 4:** Open and closed balls, Neighborhood, Open sets, Examples and basic results.

**Week 5:** Interior point, Interior of a set, Limit point, Derived set, Examples and basic results.

**Week 6:** Closed set, Closure of a set, Examples and basic results.

**Week 7:** Bounded set, Diameter of a set, Cantor’s theorem.

**Week 8:** Relativisation and subspaces, Dense sets.

**Weeks 9:** Continuous mappings, Sequential and other characterizations of continuity, Uniform

continuity,

**Week 10:** Homeomorphism, Contraction mappings, Banach fixed point theorem.

**Weeks 12 to 14:** Connectedness and compactness, Definitions and properties of connected and

compact spaces.

**Readings (in APA format)**

**Reference Book**

V Satish Shirali & Harikishan L. Vasudeva, *Metric Spaces*, Springer Verlag London

(2006) (First Indian Reprint 2009)

**Additional Readings:**

i. Kumaresan, S. (2014). *Topology of Metric Spaces* (2nd ed.). Narosa Publishing

House. New Delhi.

ii. Simmons, George F. (2004). *Introduction to Topology and Modern Analysis*.

McGraw-Hill Education. New Delhi. iii. Banerjee, A.K. & Dey, Anindya (2017) *Metric Space & Complex Analysis* (1st edition), New Age International (P) Ltd. ISBN 9789386070876 iv. Copson, E.T. (1988) *Metric Spaces* (1st paperback edition), Cambridge University press ISBN 052 35732 2 paperback

v. Jain, P.K. (2019) *Metric Spaces* (3rd edition), Narosa Publishing House, New Delhi vi. Reisel, Robert B. (1982) *Elementary Theory of Metric Spaces*, Springer-Verlag, New York, U.S.A., ISBN-13:987-0-387-90706-2

vii. Ambrosio, Luigi & Tilli, Paolo (2003) *Topics on Analysis in Metric Spaces*: *25* (Oxford Lecture Series in Mathematics & its applications) OUP Oxford, ISBN-13 978-0198529385

**Methodology of Teaching:**

1. **PPT Presentation by Students on different topics of the syllabus.**
2. **Increasing interest in the topic by organizing mathematical quiz programme.**
3. **Use of Mathematical Games on related topic to encourage thinking power of students in the topics.**
4. **Use of mathematical Software like Mathematica, MATLAB and LaTex etc.**
5. **E-resources-**

[**(PDF) INTRODUCTION TO METRIC SPACES**](https://www.researchgate.net/publication/311589083_INTRODUCTION_TO_METRIC_SPACES)

[**http://people.math.aau.dk/~fajstrup/UNDERVISNING/PHD/04/l1.pdf**](http://people.math.aau.dk/~fajstrup/UNDERVISNING/PHD/04/l1.pdf)

[**https://nptel.ac.in/courses/111105037**](https://nptel.ac.in/courses/111105037)

**(Mention the use of ICT, MOOCs fieldwork, visits, or any specific activities apart from lectures)**

**ASSESSMENT**

**Tentative date of assessments/ assignments (time frame):**

**Class Test 1 : 2nd week of September**

**Class Test 2 : 2nd week of October**

**Assignment 4th week of October**

**Class test based on Whole Syllabus on 2nd week of November**

**Criteria of Assessment: Class Test, Assignment, Presentation by students, Discussion in class. Viva in the class.**